VTrans Research

Technical Memo – Concrete Pay Factor Analysis

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From: Jim Sullivan, Dave Novak, UVM

Date: July 6, 2018

Project Scope

This memo documents the results of efforts by the UVM research team to develop an initial set of pay factors for payment of in-place concrete by the Vermont Agency of Transportation's Materials Testing & Certification Lab. The project consisted of employing a search heuristic to establish a variety of pay-incentives for synthetic data representing the assumed distributions of concrete compressive strengths (CCS) that are projected to result from the implementation of pay factors with an upper and lower acceptance boundary on 28-day CCS. The various scenario solutions were constrained by the stipulation that the final payment for concrete in the assumed synthetic distribution would total 3% more than a payment without incentive factors.

The Agency's intent is to use the new pay factor schedule to incentivize industry to change their current production and placement processes that effectively lowers the mean 28-day CCS toward the new design around a target value of 4,900 psi. The Agency currently uses only a lower acceptance bound (4,000 psi) with no variation in payment for in-place lot averages that far exceed the design target. This practice has led to an average 28-day CCS of around 7,400 psi, which is about 150% above the design target. Excessively high concrete strengths are believed to be associated with increased brittleness and excessive early cracking.

Specific direction for the scope of this analysis was received through a series of meetings between UVM researchers and VTrans Materials Testing & Certification staff on March 8, 2016, September 30, 2016, and January 13, 2017, during which it was decided that the percent-within-limits (PWLs) approach outlined in Burati et. al (2003) would be followed. Development of screening quality characteristics and screening procedures for reviewing compressive strength of in-place concrete for acceptance and payment are not included in this scope.

Initial Analysis

Constraints and Payment Incentives

To develop pay factors that would fit the constraints of the study, it was necessary to make assumptions about how industry (both contractors and producers) would likely respond to the new performance specification and payment incentives and disincentives, while balancing risk between the Agency and its contractors. The primary constraint on the scenarios was that the pay factors would result in a shift in average 28-day CCS by the Agency's contractors that would result in a net over-payment of 3% when compared to payments made without pay factors. For example, if a contractor whose concrete strengths were distributed as shown in Table 1 was paid without incentives, the total payment at $1.00 per unit would be $120, whereas the same payment with pay incentives and disincentives would be 3% larger, or $124.

Table 1 Net Over-Payment Example

Strength Range (psi)	No. of Units	Base Unit Payment	Payment without (Dis)Incentives	(Dis) Incentives	Payment with (Dis)Incentives
4000 – 4500	6	$ 1.00	$ 6.00	$ 1.00	$ 6.00
4500 – 5000	17	$ 1.00	$ 17.00	$ 1.04	$ 17.68
5000 – 5500	25	$ 1.00	$ 25.00	$ 1.08	$ 27.10
5500 – 6000	33	$ 1.00	$ 33.00	$ 1.04	$ 34.32
6000 – 6500	28	$ 1.00	$ 28.00	$ 1.00	$ 28.00
6500 – 7000	7	$ 1.00	$ 7.00	$ 0.96	$ 6.72
> 7000	4	$ 1.00	$ 4.00	$ 0.92	$ 3.68
		Totals	**$ 120**		**$ 124**

In this hypothetical example, the net overpayment of 3% (($124-$120)/$120) constrains the design of incentives. The characteristics of the pay factor incentive design that are controlled by VTrans are therefore the peak incentive (1.08) and the step reduction in incentive (0.04) as strengths move away from the peak range (5000-5500 psi).

In this project, incentives were specifically designed to maintain a net overpayment of 3% while minimizing the peak incentive. This net overpayment is incorporated into the design as an incentive for contractors to respond to the new schedule of pay factors with process adjustments that will shift the distribution of CCS into a more favorable range. In other words, the overpayment will mitigate the likelihood that contractors will elect to not change their process and simply set bid prices that absorb the expected disincentives that come from the delivery of concrete with in-place CCS well in excess of the peak incentive range. It does not guarantee that a contractor will be overpaid by 3%, but assumes that the overpayment will encourage contractors to change their processes / behaviors to capture these added payments.

Strength ranges where incentives transition to disincentives are also suggested for most of the scenarios considered. In the example shown in Table 1, the transitions occur at 4000 and 6500. The design of the incentive structure was then converted into a set of pay factors using the PWL guidance from Burati et. al., (2003).

Scenarios Considered in Designing Incentives

A variety of scenarios were considered in the process of designing incentives. Each scenario represents a different combination of: 1) projected CCS lot distributions (mean, standard deviation), and 2) incentive characteristics (upper acceptance bounds, upper and lower reward bounds, and shape of reward design distribution). The scenarios capture variations in the expected response to the implementation of different incentive strategies and represent a range of risk possibilities to both the contractor and the Agency. Our approach allows for additional constraints including an upper and lower acceptance bound, an upper and lower reward bound, a peak incentive location, and an incentives shape. The acceptance bounds correspond to upper and lower limits that define acceptable CCS. A new lot of concrete that falls outside of the acceptance bounds is considered unacceptable and will not be paid for (and must be removed).

It is important to stress that the exact changes in contractor process / behavior will not be known until after post-implementation (i.e., the new pay factors are established, contractors respond in some way or not, and the Agency collects and analyzes the new data). There are many possible scenarios that could be considered. The specific characteristics of each of the seven scenarios considered in the report were determined collaboratively by UVM researchers and Agency staff and were designed to reflect different possible approaches as well as "reasonable" CCS lot distributions. Table 2 provides a summary of the characteristics of each scenario.

Table 2 Summary of Scenarios Considered in the Design of Incentives

	Scenarios						
	A	**F**	**G**	**H**	**F-a**	**F-b**	**F-c**
New Lot Distribution Characteristics (psi)							
Average or Peak	7,400	4,900	7,400	4,900	4,900		
Standard Deviation	1,000		-	-	500	250	1,000
Shape	Historical	Historical – 2,500	Historical	Historical – 2,500 Inverse	Symmetric (Normal)		Historical – 2,500 Inverse
Incentives Design Characteristics (psi)							
Lower Acceptance Bound	4,000						
Upper Acceptance Bound	6,000	8,000	11,000	11,000	8,000	8,000	8,000
Lower Reward Bound	4,000	4,300	4,000	4,000	4,300	4,300	4,000
Upper Reward Bound	6,000	5,700	9,500	9,500	5,500	5,500	8,500
Peak Incentive Location	4,900						
Shape	Symmetric		Asymmetric (Long Tail)		Symmetric		

Scenarios A and G assume that there will be no dramatic responses to the pay factors. Thus, the new CCS averages will be identical to the historical lot averages. Scenario F assumes that the average 28-day CCS of the new in-place concrete will be uniformly reduced by 2,500 psi, but all other characteristics of the distribution will remain the same, as illustrated in Figure 1.

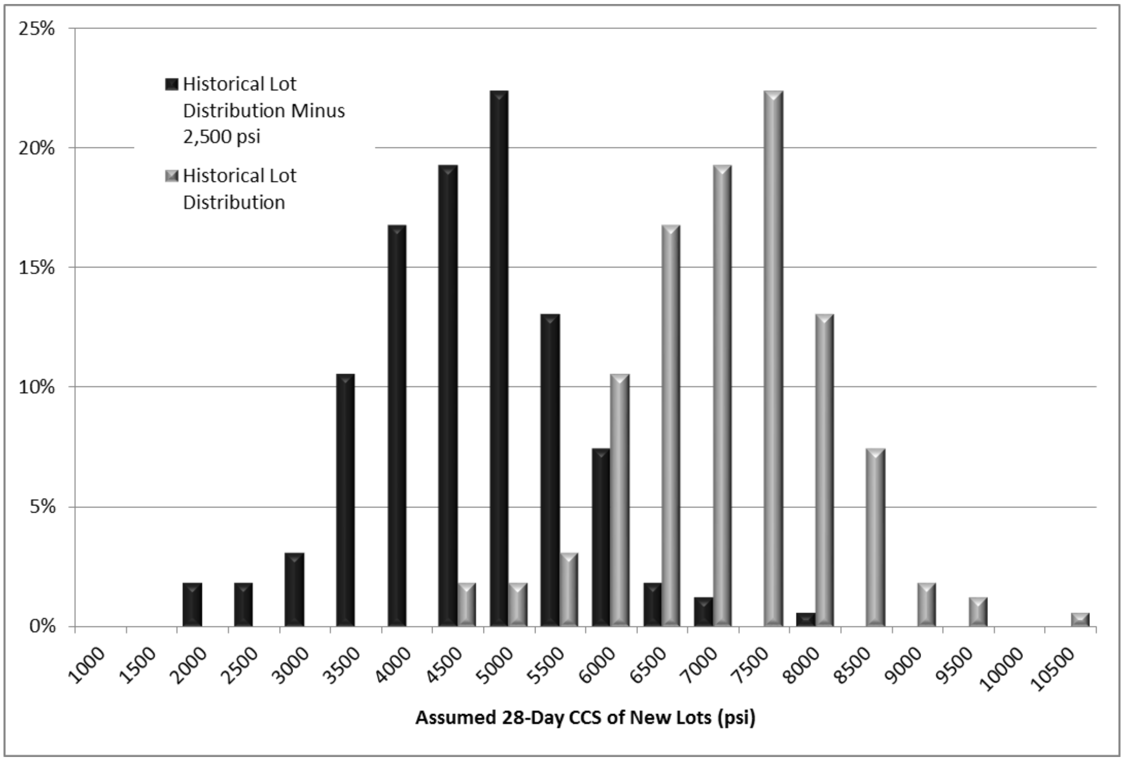

Figure 1 Assumed Distribution of New 28-Day CCS

Scenarios H and F-c assume an asymmetrical response to the pay factors and result in the creation of a distribution of new 28-day CCS that is the inverse of the historical distribution minus 2,500 psi. This distribution of new lot averages can be approximated by a gamma distribution with a peak at 4,900 psi. Scenarios F-a and F-b assume a synthetic distribution of new 28-day CCS with standard deviations that are smaller than the historical averages by ½ and ¼ respectively.

The incentive structure associated with each of the seven scenarios provides a net 3% overpayment and a “constrained” peak incentive, so that the incentive structure can be “spread” over a range of CCS values as opposed to being narrowly focused in a small range. For example, it is possible to design an incentive structure that only provides payment incentives to lots falling between 4,500 – 5,500 psi. However, if it is too costly or difficult for contractors to consistently produce lots where the average CCS falls in that range, they may not even try to do so. The upper and lower reward bounds define an area around the peak incentive where pay factors are greater than 1.0 – representing situations where the contractor is rewarded. Outside of the reward bounds (but inside the acceptance bounds), lots are accepted but the contractor is penalized with a pay factor less than 1.0. For all of the scenarios, the location of the peak incentive is 4,900 psi. This value represents the ideal in-place 28-day CCS. Most of the incentive structures are designed with a symmetric shape, meaning that the incentives above the peak are identical to those an equal distance below the peak. This shape can hold even when the lower and upper acceptance bounds are different. The shape of the incentives distribution for Scenario F-a is shown in Figure 2 to illustrate these constraints.

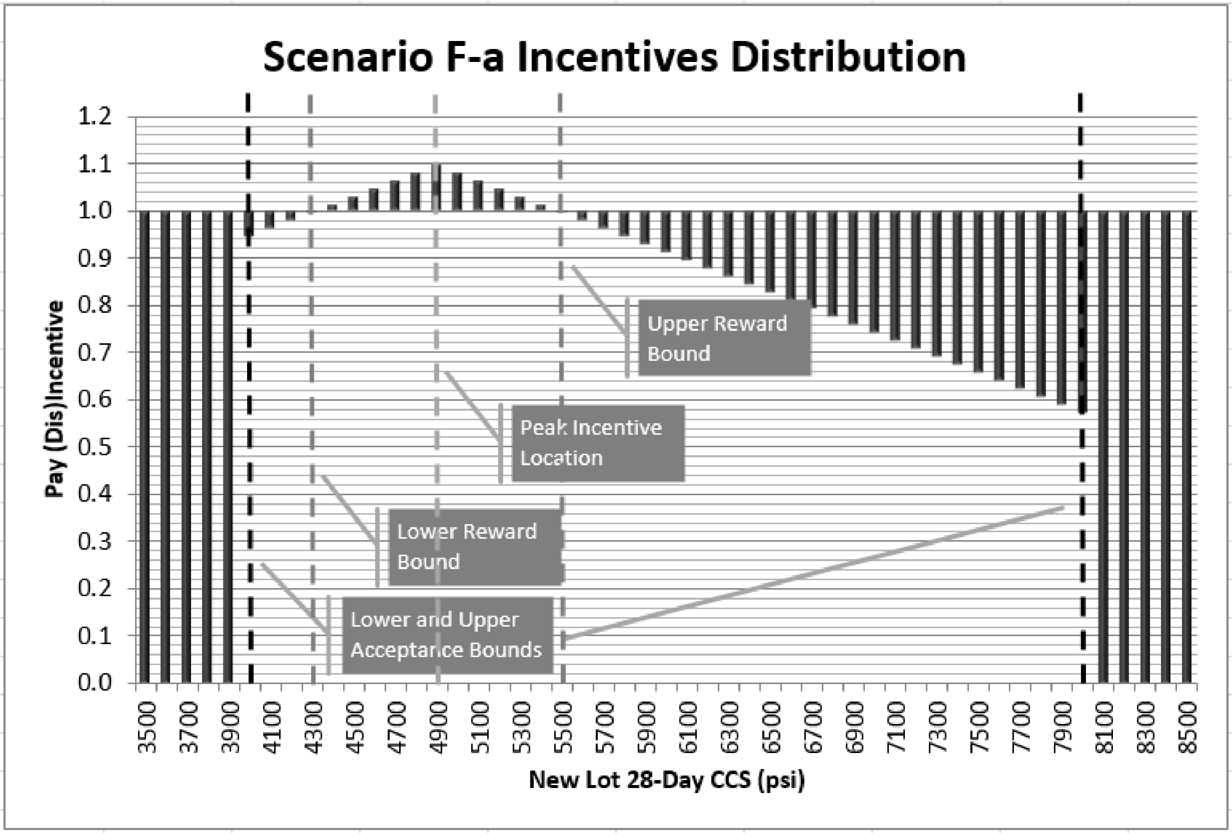

Figure 2 Incentives distribution for Scenario F-a

Converting Pay Incentives to "Percent Within Limits"

Once a distribution of pay incentives is designed, the pay incentives need to be converted to a schedule of pay factors that correspond to the percent-within-limits (PWLs) of the new lot being tested. The PWL approach allows for the use of averaged values of new lot 28-day CCS for a variety of *n* values. Following the guidance of Burati et. al., (2003), PWL values are assigned to an average new lot 28-day CCS based on its distance from the upper or lower acceptance bounds relative to the standard deviation of the sample, and follows a two-step process. First, the following formulas are used to calculate a Quality Index (*Q*) for the mean of the sample (*X*):

$$Q_L = \frac{X - LAB}{s}$$

$$Q_U = \frac{UAB - X}{s}$$

where *LAB* and *UAB* are the lower and upper acceptance bounds respectively. Second, the Quality Indices (Q_L and Q_U) are used in a lookup table to identify the PWLs the correspond to each of the Quality Indices depending on the number of samples *n* used to calculate the sample mean. The lookup table used for this project is provided in Attachment A.

One of the contributions of this research is the creation of an upper bound for CCS and the utilization of both an upper and a lower acceptance bound. Historically, only a lower bound is used, so even lots that far exceed the mean CCS target are accepted. To implement the upper and lower acceptance bound approach, the two different PWLs are aggregated to produce a single PWL value to use in the pay factor table. We suggest an aggregation that involves subtracting each PWL_U and PWL_L from 100 to produce the percent defective (PD) for each PWL (PD_U and PD_L). The two PDs are summed and then subtracted from 100 to yield the new aggregate PWL as follows:

$$PWL = 100 - [(100 - PWL_U) + (100 - PWL_L)]$$

or

$$PWL = 100 - [(PD_U + PD_L)]$$

Incentive Design Results

Scenarios G and H illustrate how pay factors might be developed if the incentives are structured asymmetrically. In these cases, incentives fall off quickly for psi strengths below the target incentive value, but more slowly for psi strengths above the target incentive value. The gradual reduction of the incentives for strengths above the mean is consistent with observed behaviors resulting from the fact that, historically, contractors are not faced with an upper acceptance bound. While both the research team and the VTrans Material Lab personnel agreed that the creation and use of asymmetrical pay factors could potentially be useful, pay incentives with a non-normal distribution represent a somewhat "radical" departure from well-established PWL calculation methods using the guidance provided Burati et. al., (2003). The desire to base the new pay factors on the well-known and widely accepted PWL method made the development of asymmetric incentive scenarios less appealing.

Scenario A assumes that the contractors will not respond to the implementation of pay incentives, but the upper acceptance bound will be at 6,000 psi, implying that a large portion of the lots will be rejected. In this scenario, relatively few samples are accepted, and those are rewarded highly in order to counteract the high rate of rejection. The end result is a net overpayment of 1.03. With an upper acceptance bound of 6,000 psi, it is unlikely that the contractors will perform similarly to the historical mean in-place CCS of 7,400 psi. Therefore, this scenario was determined to be unrealistic.

Scenario F allows for a response by the contractors where the mean of the new lot distribution drops to the target of 4,900 psi and includes a symmetrical pay incentive structure with a more realistic upper acceptance bound of 8,000 psi. However, given a standard deviation of 1,000 psi, the new lot distribution often falls below the lower acceptance bound of 4,000 psi, yielding many rejected lots. This scenario was also determined to be slightly unrealistic.

Scenarios F-a, F-b, and F-c are more realistic. All scenarios assume both a change in the mean of the new lot distribution to 4,900 psi. Scenarios F-a and F-b also assume a reduction in the standard deviation to 500 psi and 250 psi respectively. Scenario F-c assumes the standard deviation remains at 1,000 psi, but that the shape of the CCS changes in response to the pay factors. The results associated with Scenarios F-a, F-b, and F-c are shown in Figures 3a-c.

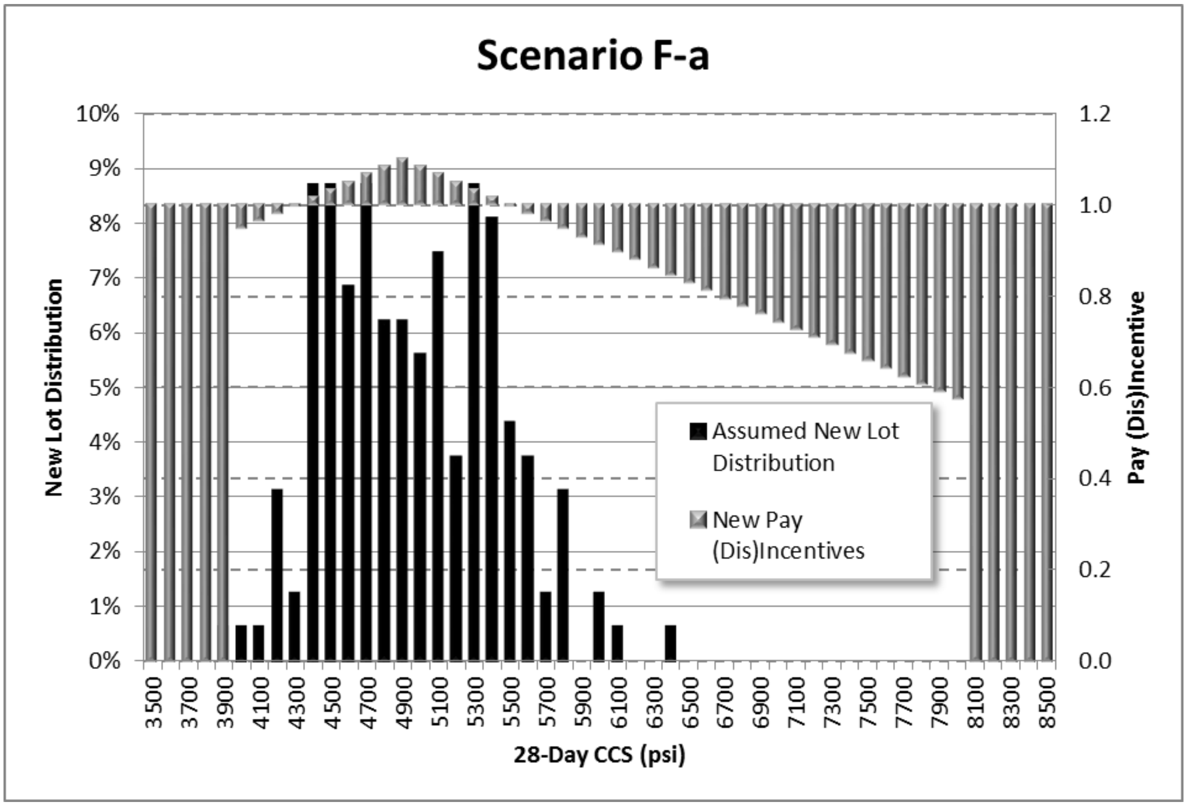

Figure 3a Pay Incentive Design for Scenario F-a

Scenario F-a provides a set of pay incentives that equally balance the risk between the industry and the Agency. The assumption that the standard deviation of the new lot distribution will decrease to 500 psi is justified by a relatively small margin of error (900 psi) between the lower acceptance bound and the peak incentive location. Less than 1% of the assumed new lots are rejected under Scenario F-a, and the peak incentive of 1.10 offers the industry an enticing reward for hitting the peak incentive of 4,900 psi. Risk to the Agency is limited, as it seems unlikely that the new lot distribution will be focused tightly enough around the peak incentive to create a large likelihood of net overpayment.

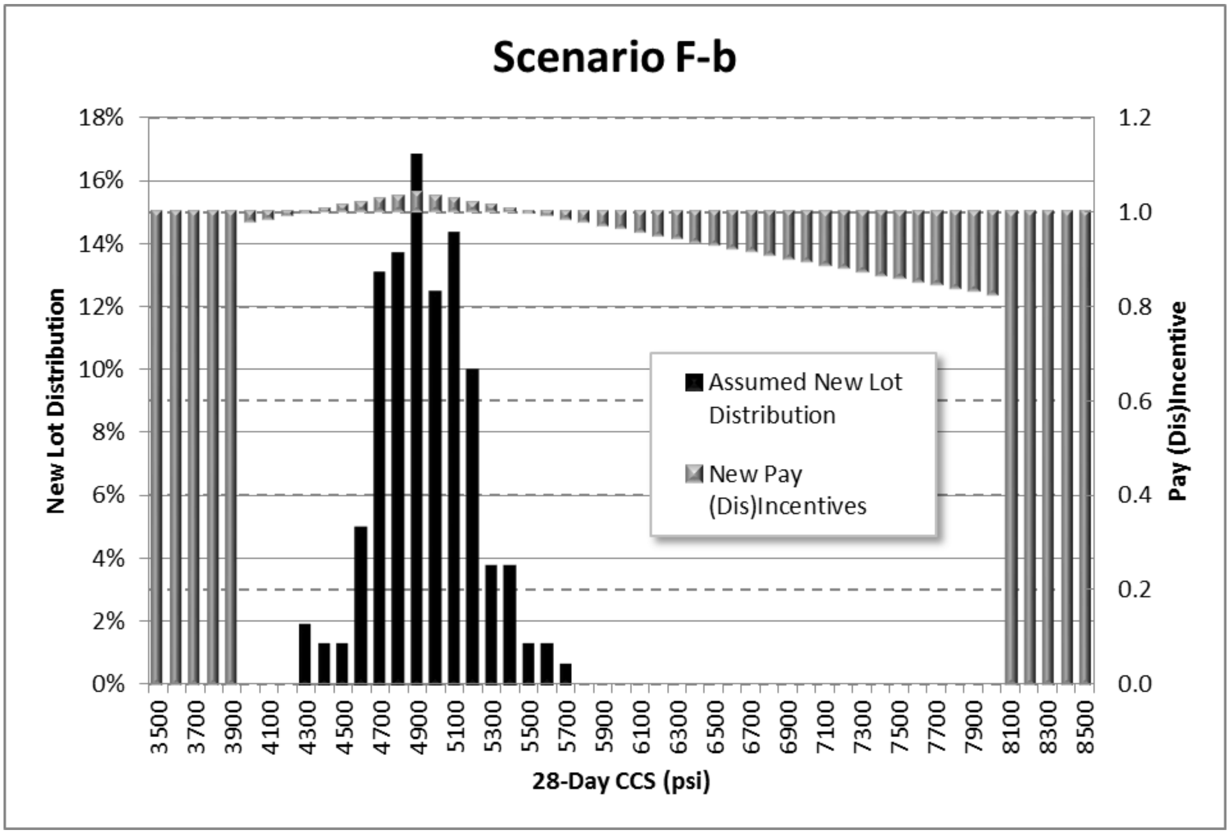

Figure 3b Pay Incentive Design for Scenario F-b

Scenario F-b assumes a smaller standard deviation (250 psi) for the new lot distribution, with results clustered tightly around the peak incentive location and no rejected lots. With all of the lots accepted and clustered tightly around 4,900 psi, the incentive structure is designed to produce very limited pay incentives. In this case, almost all of the lots are rewarded, with peak incentive of 1.01 and a maximum penalty of only 0.98. This scenario was considered to be overly idealistic and possibly represents a stage the industry and the Agency will reach over a period of time after more significant incentives have created a shift in the industry to methods that are capable of producing very precise 28-day CCS clustered around 4,900 psi.

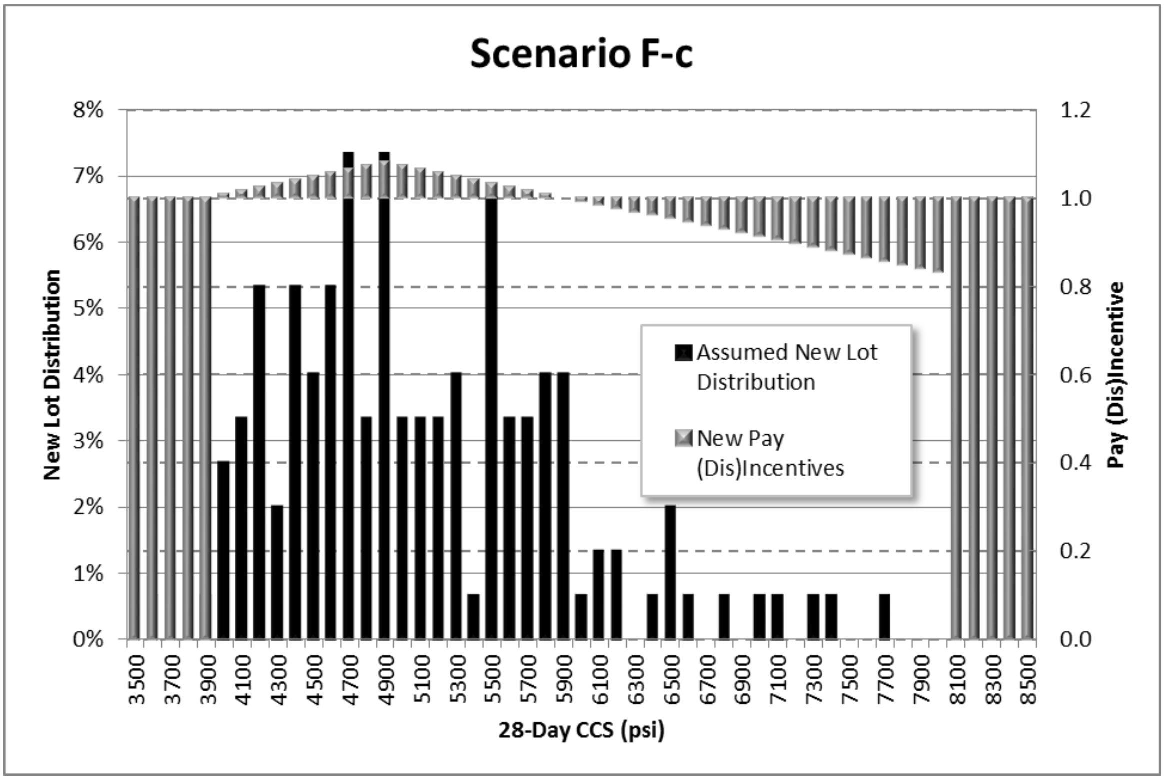

Figure 3c Pay Incentive Design for Scenario F-c

Scenario F-c assumes the same standard deviation as the historical results (1,000 psi). In this scenario, 7% of the new lots fall under the lower acceptance bound and another 19% of the new lots being rejected because the corresponding PWL is lower than 50% (even though the lots might be within the upper acceptance bound). Additionally, all of the acceptable CCS lots earn a pay reward. This scenario is considered to be risky to the Agency in terms of the likelihood of paying in excess of the target 3% if the industry is able to make the necessary adjustments needed to reduce the standard deviation of the new lots.

Percent-Within Limits (PWL) Results

Converting the pay incentives distributions for Scenarios F-a, F-b, and F-c into a schedule of pay factors yields the following results for 70-200 samples in the new lot distribution (see Table 3):

Table 3 Example Pay Factors for Scenarios F-a, F-b, and F-c

Scenario	PWL	Pay Factor
F-a	97-100	1.10
	95-96	1.08
	92-94	1.07
	89-92	1.05
	85-88	1.03
	80-84	1.02
	74-79	1.00
	67-73	0.98
	58-66	0.96
	50-57	0.95
F-b	95-100	1.01
	89-94	1.00
	80-88	0.99
	50-79	0.98
F-c	82-100	1.08
	80-81	1.07
	74-79	1.06
	69-73	1.05
	67-68	1.04
	63-66	1.03
	54-62	1.02
	50-53	1.01

These results confirm our earlier hypotheses that F-c puts the Agency at undue risk and F-b does not contain a strong enough incentive for the industry to make notable changes. F-a balances the risk between the Agency and the industry and provides industry with a reasonably strong incentive to change. Based on these results, we recommend Scenario F-a. Table 4 contains expanded results for PWLs for Scenario F-a for a variety of *n* values in the new lot distribution:

Table 4 Recommended Pay Factor Table

	PWLs >= (for n=)														
Pay Factor	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10-11**	**12-14**	**15-18**	**19-25**	**26-37**	**38-69**	**70+**	**Q**
1.08	100	100	98	97	96	96	96	95	95	95	95	95	95	95	1.60
1.07	100	98	95	94	93	93	93	93	92	92	92	92	92	92	1.40
1.06	100	93	92	91	91	91	91	91	91	91	91	91	90	90	1.30
1.05	100	90	90	89	89	89	89	89	89	89	89	89	89	89	1.20
1.04	90	87	87	87	87	87	87	87	87	87	87	87	86	86	1.10
1.03	83	84	84	84	84	84	84	84	84	84	84	84	84	84	1.00
1.02	74	77	77	78	78	79	79	79	79	79	79	79	79	79	0.80
1.01	71	73	74	75	75	75	75	75	75	76	76	76	76	76	0.70
1.00	67	70	71	71	72	72	72	72	72	72	72	72	72	73	0.60
0.99	64	67	68	68	68	68	69	69	69	69	69	69	69	69	0.50
0.98	61	63	64	65	65	65	65	65	65	65	65	65	65	65	0.40
0.97	58	60	61	61	61	61	61	61	62	62	62	62	62	62	0.30
0.96	56	57	57	57	57	57	58	58	58	58	58	58	58	58	0.20
0.95	50	50	50	50	50	50	50	50	50	50	50	50	50	50	0.00
Reject	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

Initial Summary and Recommendation

Based on our analysis and the endorsement of Scenario F-a, a pay factor schedule based on the PWLs is recommended. To implement the pay factor schedule, calculation of quality indices is first required. The calculations are performed separately for the lower acceptance boundary (LAB) and the upper acceptance boundary (UAB). The LAB and UAB for the recommended pay factor schedule (Scenario F-a) are 4,000 psi and 8,000 psi, respectively. The formulae for calculating the upper and lower quality indices (Q) are:

$$Q_L = \frac{\bar{X} - 4{,}000}{s}$$

$$Q_U = \frac{8{,}000 - \bar{X}}{s}$$

Where $\bar{X}$ is the mean CCS of the lot and s is the standard deviation of the lot, calculated as:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}}$$

and n is the number of individual CCS tests included in the lot.

The Q statistics represent the distance that the sample mean is offset from the specification limit. A positive Q represents the number of sample standard deviation units that the sample mean falls *inside* the specification limit, whereas a negative Q represents the number of sample standard deviation units that the sample mean falls *outside* the specification limit.

The Q_U and Q_L values are then used to look up PWL_U and the PWL_L using a set of standard tables relating quality indices to PWLs. We recommend using the lookup Table 5 in Burati et. al. (2003), which comes directly from FHWA Technical Advisory T5080.12, *Specification Conformity Analysis* (FHWA, 1989). This table is provided in Attachment A. Once the PWL_U and the PWL_L values are determined, the total sample PWL is found using as follows:

$$PWL = 100 - [(100 - PWL_U) + (100 - PWL_L)]$$

The single PWL value is then used to determine the pay factor for the lot using the PWL Table for the appropriate number of samples (*n*):

Table 5 Recommended PWL Table

	PWLs >= (for n=)							
Pay Factor	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10 or 11**
1.08	100	100	98	97	96	96	96	95
1.07	100	98	95	94	93	93	93	93
1.06	100	93	92	91	91	91	91	91
1.05	100	90	90	89	89	89	89	89
1.04	90	87	87	87	87	87	87	87
1.03	83	84	84	84	84	84	84	84
1.02	74	77	77	78	78	79	79	79
1.01	71	73	74	75	75	75	75	75
1.00	67	70	71	71	72	72	72	72
0.99	64	67	68	68	68	68	69	69
0.98	61	63	64	65	65	65	65	65
0.97	58	60	61	61	61	61	61	61
0.96	56	57	57	57	57	57	58	58
0.95	50	50	50	50	50	50	50	50
Reject	0	0	0	0	0	0	0	0

In addition to making contractors aware of this procedure for calculating pay factors the following points should be made clear:

- Lot averages outside of the acceptance boundaries (4,000 to 8,000 psi) will result in PWLs lower than 50%, and the lot will be rejected.
- Lot averages close to the Agency's target level of 4,900 psi with low standard deviations will result in the highest pay factors.
- Using this set of pay factors, the lower and upper limits for a pay factor greater than 1.00 are 4,300 and 5,500 psi, respectively.
- In using the lookup tables (in Attachment A and above) interpolation or approximation should be used if necessary to find the exact quality indices (Q) or PWLs.

Example Pay Factor Determination
We next demonstrate the process for determining a pay factor for a hypothetical lot of CCSs.

> *Assume a contractor collects 5 samples and tests the 28-day CCS of each, yielding the following results (in psi): 4,620; 5,140; 5,510; 7,480; 7,720.*
>
> *The mean CCS of the lot is (4,620 + 5,140 + 5,510 + 7,480 + 7,720)/5 =* ***6,094 psi***
>
> *The standard deviation of the lot is the square root of* $((4620 - 6094)^2 + (5140 - 6094)^2 + (5510 - 6094)^2 + (7480 - 6094)^2 + (7720 - 6094)^2)/5$ = ***1,413 psi***
>
> *Using the mean and standard deviation above, we calculate the lower quality index (Q_L) (6,094 – 4,000)/1,413 =* ***1.48*** *and the upper quality index (Q_U) (8,000 – 6,094)/1,413 =* ***1.35***
>
> *We then look up the Q_L and Q_U for a sample of "n = 5" using the table in Attachment A (Quality Index Values for Estimating PWL) by moving down the column to the value nearest the Q statistic being sought. In this case (n = 5), a Q_U value of 1.35 appears exactly at the row corresponding to a PWL_U of* ***93%****. For the Q_L, the value of 1.48 doesn't appear in the column, but the closest value is 1.49, which appears in the row corresponding to a PWL_L of* ***96%****.*
>
> *Using these values, the total sample PWL is calculated as 100 – ((100 – PWL_U) + (100 – PWL_L)) =* ***89%***
>
> *The pay factor for this lot is then found by looking up the PWL in the "PWL Table" above. Enter the table at the column corresponding to "n = 5" and follow the column down to locate a PWL of 89%. Although the PWL value of 89% doesn't appear in this column, it is closest to the value of 90% (because the value below it is 87%, which is farther away). The pay factor corresponding to the row with the 90% PWL is* ***1.05****, meaning that the quantity of material represented by these samples is paid at a rate 5% higher than the base bid price.*

Secondary Analysis
This memo documents the results of a secondary analysis by UVM researchers to develop a final set of pay factors for payment of in-place concrete that is responsive to the concerns of the industry representatives. The scope of this secondary analysis resulted from a meeting with the industry representatives in December of 2017 in which the primary set of pay factors was presented and comments were received. Specific direction for the scope of this secondary analysis was received in a meeting between Jim Sullivan and Nick Van Den Berg on March 27, 2018. The percent-within-limits (PWLs) approach outlined in Burati et. al (2003) will be followed, and the final secondary scenario would be developed with the following constraints:

- Target design mean = 5,000 psi
- Lower acceptance boundary = 3,500 psi
- Lower reward boundary = 4,000 psi
- "Dummy" upper acceptance boundary = 6,500 psi
- Upper reward boundary = 6,000 psi
- Simulated industry response: $\mu \approx$ 5,000 psi; SD $\approx$ 500 psi
- Net overpayment = 3%
- Incentive factor for lots outside the reward boundaries will decline linearly from 1.00 to 0.80 at the acceptance boundary
- Incentive factor for lots below 3,500 will be 0.00.
- Incentive factor for lots above 6,500 will be 0.80 (hence the term "Dummy" above)

Incentive factors inside the reward boundaries will be designed using the same methods used previously. Incentive factors outside the reward boundaries will not follow the same rules as the initial analysis, but will be fixed at a pre-determined constant.

There was also an interest in removing the sharp drop in the pay factors that resulted in previous scenarios between the target design mean and the upper acceptance boundary. To accomplish this, the PWL method will be applied as if the upper acceptance boundary was 6,500.

Secondary Incentives Design Results

The Secondary Analysis yielded a set of pay incentives that balance the risk between the industry and the Agency (Figure 4).

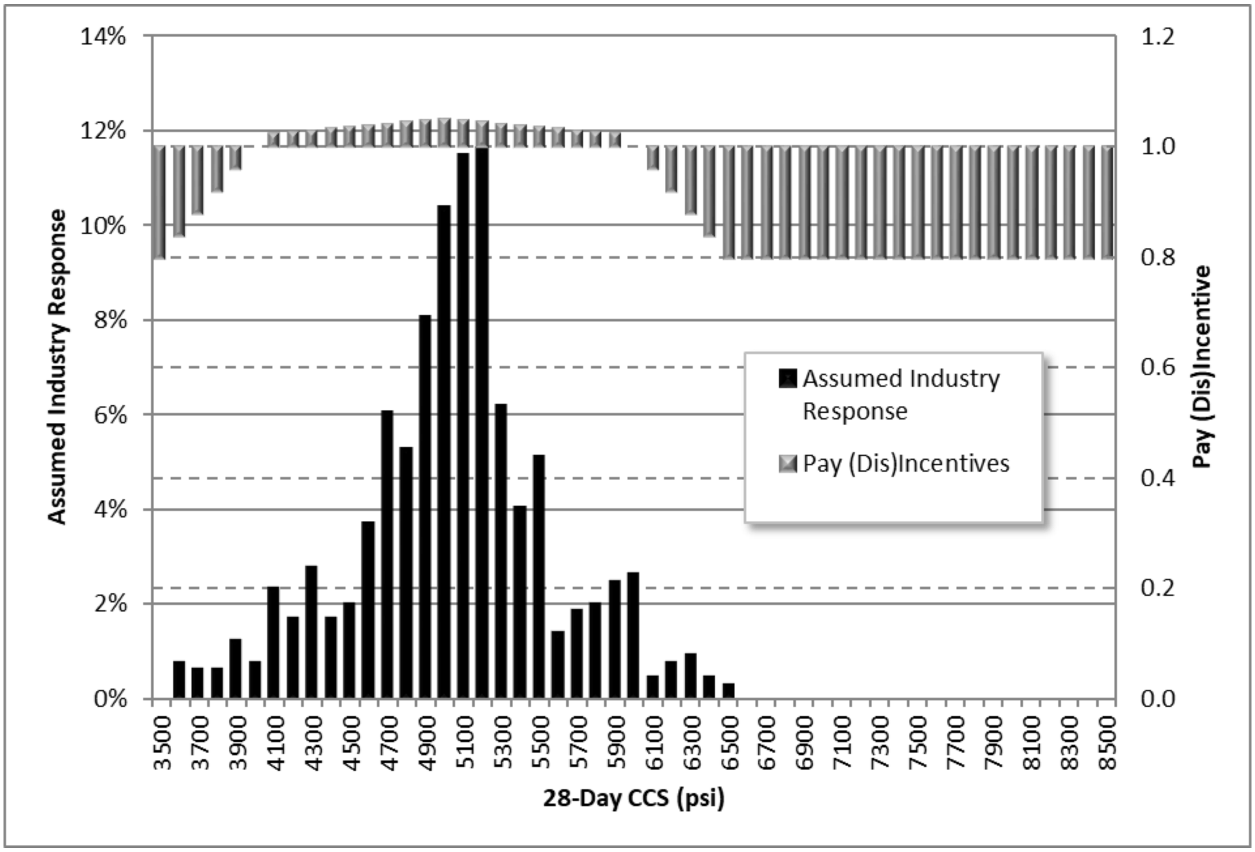

Figure 4 Pay Incentive Design for Secondary Analysis

None of the assumed new lots are rejected and the peak incentive of 1.05 offers the industry a reward for hitting the peak incentive at 5,000 psi. However, it seems unlikely that the new lot distribution will be focused tightly enough around the peak incentive to create a net overpayment exceeding 3%.

A set of pay incentives was also calculated using a peak incentive of 1.04, but it contained no variation in pay incentive within the reward boundary, with all incentives equal to the peak (1.04). Other solutions were also found with peak incentives of 1.06 and 1.07, but these were determined to put the Agency at undue risk.

Percent-Within Limits Results and Recommended Pay Factors

Converting the recommended pay incentives into a schedule of pay factors yields the results shown in Table 6 for a variety of *n* values and means in the new lot distribution.

Table 6 Relationship Between Q Statistic, PWL and Pay Factor

Avg. Strength of the Lot	Q	PWLs (for a lot with n=)										Pay Factor
		3	4	5	6	7	8	9	10	19-25	70-200	
3500	0.00	50	50	50	50	50	50	50	50	50	50	0.80
3600	0.20	56	58	57	58	59	59	59	59	58	58	0.84
3700	0.40	62	64	64	66	66	65	65	65	66	67	0.88
3800	0.60	68	70	71	72	73	72	72	72	73	74	0.92
3900	0.80	76	78	78	78	78	79	79	79	79	80	0.96
4000	1.00	83	84	84	85	84	84	84	84	84	85	1.00
4100	1.20	100	90	90	89	89	89	89	89	89	89	1.02
4200	1.40	100	98	95	94	93	93	93	93	92	92	1.03
4300	1.60	100	100	98	97	96	96	96	95	95	95	1.03
4400	1.80	100	100	100	99	98	98	98	98	97	97	1.03
4500	2.00	100	100	100	100	99	99	99	99	98	98	1.04
4600	2.20	100	100	100	100	100	99	99	99	99	98	1.04
4700	2.40	100	100	100	100	100	100	99	99	99	99	1.04
4800	2.60	100	100	100	100	100	100	100	99	99	99	1.04
4900	2.80	100	100	100	100	100	100	100	100	99	99	1.05
5000	3.00	100	100	100	100	100	100	100	100	100	99	1.05
5100	2.80	100	100	100	100	100	100	100	100	99	99	1.05
5200	2.60	100	100	100	100	100	100	100	99	99	99	1.04
5300	2.40	100	100	100	100	100	100	99	99	99	99	1.04
5400	2.20	100	100	100	100	100	99	99	99	99	98	1.04
5500	2.00	100	100	100	100	99	99	99	99	98	98	1.04
5600	1.80	100	100	100	99	98	98	98	98	97	97	1.03
5700	1.60	100	100	98	97	96	96	96	95	95	95	1.03
5800	1.40	100	98	95	94	93	93	93	93	92	92	1.03
5900	1.20	100	90	90	89	89	89	89	89	89	89	1.02
6000	1.00	83	84	84	85	84	84	84	84	84	85	1.00
6100	0.80	76	78	78	78	78	79	79	79	79	80	0.96
6200	0.60	68	70	71	72	73	72	72	72	73	74	0.92
6300	0.40	62	64	64	66	66	65	65	65	66	67	0.88
6400	0.20	56	58	57	58	59	59	59	59	58	58	0.84
6500	0.00	50	50	50	50	50	50	50	50	50	50	0.80
6600→8500	0.00	50	50	50	50	50	50	50	50	50	50	0.80

In Table 6, the Q indices were calculated using the lot mean shown with an assumed standard deviation of 500 psi.

Final Summary and Recommendation

A secondary set of pay factors was calculated to assess payment of in-place concrete that is responsive to the concerns of the industry representatives who convened in December 2017. This secondary set of pay factors is simpler to assess, and decreases risk to the industry even further than the recommended initial set of pay factors.

To assess the secondary set of pay factors, lots whose average is below 3,500 or above 8,500 are rejected. Lots whose average is between 6,500 and 8,500 are assessed a pay factor of 0.80. Calculation of a quality index is required for lots whose average CCS is between 3,500 and 6,500. The formula for calculating the quality index (Q) depends on the average of the lot. The formula when the lot average is under 5,000 psi is:

$$Q = \frac{\bar{X} - 3{,}500}{s}$$

The formula when the lot average is 5,000 psi or above is:

$$Q = \frac{6{,}500 - \bar{X}}{s}$$

Where $\bar{X}$ is the mean CCS of the lot and s is the standard deviation of the lot, calculated as:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}}$$

n is the number of individual CCS samples included in the lot.

The Q statistic represents the distance that the mean is offset from the target design mean. The Q value is then used to look up the PWL using a set of standard tables relating quality indices to PWLs for a given number of samples n. We recommend using the lookup table "Quality Index Values for Estimating PWL" in Burati et. al. (2003). The Q statistic or the PWL value can then be used to determine the pay factor for the lot using Table 7.

Table 7 Pay Factor Table

Q	PWLs (for a lot with n=)										Pay Factor
	3	4	5	6	7	8	9	10	19-25	70-200	
0.00	50	50	50	50	50	50	50	50	50	50	0.80
0.20	56	58	57	58	59	59	59	59	58	58	0.84
0.40	62	64	64	66	66	65	65	65	66	67	0.88
0.60	68	70	71	72	73	72	72	72	73	74	0.92
0.80	76	78	78	78	78	79	79	79	79	80	0.96
1.00	83	84	84	85	84	84	84	84	84	85	1.00
1.20	100	90	90	89	89	89	89	89	89	89	1.02
1.40	100	98	95	94	93	93	93	93	92	92	1.03
1.60	100	100	98	97	96	96	96	95	95	95	1.03
1.80	100	100	100	99	98	98	98	98	97	97	1.03
2.00	100	100	100	100	99	99	99	99	98	98	1.04
2.20	100	100	100	100	100	99	99	99	99	98	1.04
2.40	100	100	100	100	100	100	99	99	99	99	1.04
2.60	100	100	100	100	100	100	100	99	99	99	1.04
2.80	100	100	100	100	100	100	100	100	99	99	1.05
3.00	100	100	100	100	100	100	100	100	100	99	1.05

In addition to making industry aware of this procedure for calculating pay factors, the following points should be made clear:

- Lot averages close to the Agency's target level of 5,000 psi with low standard deviations will result in the highest pay factor (1.05).
- Using this set of pay factors, the lower and upper limits for a pay factor greater than 1.00 are 4,000 and 6,000 psi, respectively, if the target standard deviation of 500 psi is achieved.
- In using the lookup tables (in Attachment A and above) interpolation or approximation should be used if necessary, to find the exact quality indices (Q) or PWLs.

References

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